

Consider the conic with equation $3x^2 - 2y^2 + 6x + 12y - 3 = 0$.

SCORE: ____ / 30 PTS

- [a] Find the co-ordinates of the focus/foci.

$$3x^2 + 6x - 2y^2 + 12y = 3$$

$$3(x^2 + 2x + 1) - 2(y^2 - 6y + 9) = 3 + 3 - 18$$

$$3(x+1)^2 - 2(y-3)^2 = -12$$

$$\begin{array}{c} \smile \\ \cdot \\ \smile \end{array} \quad \frac{(y-3)^2}{6} - \frac{(x+1)^2}{4} = 1 \quad c^2 = 6 + 4 = 10$$

$$(-1, 3 \pm \sqrt{10})$$

- [b] If the equation corresponds to a circle, find its radius.

If the equation corresponds to a parabola, find its vertex & directrix.

If the equation corresponds to an ellipse, find the endpoints of its minor axes.

If the equation corresponds to a hyperbola, find the equations of the asymptotes.

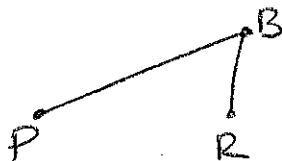
$$y-3 = \pm \frac{\sqrt{6}}{2}(x+1)$$

Pat & Reese live in separate houses on Taylor Street.
Taylor Street is parallel to Jordan Road. Both roadways are straight.

SCORE: ____ / 10 PTS

Draw diagrams and write algebraic equations involving distances to answer the following questions.

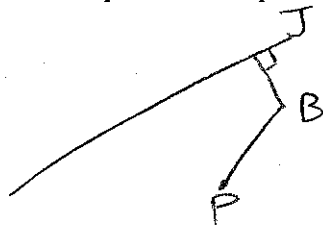
- [a] A bird is flying through the neighborhood. The distance from Pat's house to the bird to ^{REESE}Chris's house is always 300 meters.
What is the shape of the bird's path? (Assume the bird is flying at a constant low height.)



$$BP + BR = 300$$

ELLIPSE

- [b] There is a bike path nearby. Every point on the path is half as far from Jordan Road as it is from Pat's house.
What is the shape of the bike path?



$$BJ = \frac{1}{2} BP$$

$$e = 2 = \frac{BP}{BJ}$$

HYPERBOLA

Consider the ellipse with foci $(-7, -2)$ and $(-7, 4)$ and minor axis of length 12.

SCORE: ____ / 20 PTS

[a] Find the equation of the ellipse.

$$\text{CENTER} = (-7, \frac{-2+4}{2}) = (-7, 1)$$

$$2b = 12 \rightarrow b = 6$$

$$c = 4 - 1 = 3$$

$$a^2 = b^2 + c^2 = 45$$

$$\cdot F(-7, 4)$$

$$\cdot C(-7, 1)$$

$$\cdot F(-7, -2)$$

$$\frac{(x+7)^2}{6^2} + \frac{(y-1)^2}{45} = 1 \rightarrow \frac{(x+7)^2}{36} + \frac{(y-1)^2}{45} = 1$$

[b] Find the co-ordinates of the vertices.

$$(-7, 1 \pm 3\sqrt{5})$$

Convert the rectangular equation $y = 2x - 3$ to polar.

SCORE: ____ / 15 PTS

$$r \sin \theta = 2r \cos \theta - 3$$

$$r \sin \theta - 2r \cos \theta = -3$$

$$r(\sin \theta - 2 \cos \theta) = -3$$

$$r = - \frac{3}{\sin \theta - 2 \cos \theta}$$

$$r = \frac{3}{2 \cos \theta - \sin \theta}$$

Consider the polar equation $r = -2 - 3\sin\theta$.

SCORE: ____ / 30 PTS

The following symmetry tests all fail: $(-r, \theta)$, $(-r, -\theta)$ and $(-r, \pi - \theta)$

[a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

POLAR AXIS: $(r, -\theta)$

$$r = -2 - 3\sin(-\theta)$$

$$r = -2 + 3\sin\theta$$

SYMMETRIC OVER POLAR AXIS?	NO CONCLUSION
SYMMETRIC OVER POLE?	NO CONCLUSION
SYMMETRIC OVER $\theta = \frac{\pi}{2}$?	YES

POLE: $(r, \pi + \theta)$

$$r = -2 - 3\sin(\pi + \theta)$$

$$r = -2 - 3[\sin\pi\cos\theta + \cos\pi\sin\theta]$$

$$r = -2 + 3\sin\theta$$

$\theta = \frac{\pi}{2}$: $(r, \pi - \theta)$

$$r = -2 - 3\sin(\pi - \theta)$$

$$r = -2 - 3[\sin\pi\cos\theta - \cos\pi\sin\theta]$$

$$r = -2 - 3\sin\theta$$

[b] What is the minimum interval of θ - values that must be plotted before using symmetry to complete the graph?

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

[c] Name the shape of the graph of the polar equation. If the graph is a rose curve, state the number of petals.

$$|-2| < |-3| \rightarrow \text{LIMACON WITH LOOP}$$

Find the equation of the parabola with focus $(15, -17)$ and directrix $y = 11$.

SCORE: ____ / 15 PTS

~~_____~~ $y = 11$
 $p = -14 \downarrow$
 $V(15, -3)$
 $F(15, -17)$

$$\text{VERTEX} = \left(15, \frac{-17+11}{2}\right) = (15, -3)$$

$$p = -17 - (-3) = -14$$

$$(x-15)^2 = 4(-14)(y+3)$$

$$(x-15)^2 = -56(y+3)$$

Consider the conic with the polar equation $r = \frac{36}{7-5\cos\theta}$.

SCORE: ____ / 30 PTS

- [a] What is the shape of the graph of the equation?

$$r = \frac{\frac{36}{7}}{1 - \frac{5}{7}\cos\theta} \quad e = \frac{5}{7} < 1 \rightarrow \text{ELLIPSE}$$

- [b] Find the equation of the directrix.

$$\begin{aligned} ep &= \frac{36}{7} \\ \frac{5}{7}p &= \frac{36}{7} & x &= -\frac{36}{5} \\ p &= \frac{36}{5} \end{aligned}$$

- [c] Find the rectangular co-ordinates of the focus/foci, using the process in the lecture & website handout.

θ	r	(x, y)
0	18	(18, 0)
$\frac{\pi}{2}$	$\frac{36}{7}$	$(0, \frac{36}{7})$
π	3	(-3, 0)
$\frac{3\pi}{2}$	$\frac{36}{7}$	$(0, -\frac{36}{7})$

← VERTICES

CENTER = $(\frac{18-3}{2}, 0)$
= $(\frac{15}{2}, 0)$

FOCI = (15, 0),
(0, 0)